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ACCELERATED GENETIC ALGORITHM SOLUTION OF LINEAR BLACK – SCHOLES EQUATION

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ABSTRACT

In this research, the development of a fast numerical method was performed in order to solve the Option Pricing problems governed by the Black-Scholes equation using an accelerated genetic algorithm method. Where the Black-Scholes equation is a well known partial differential equation in financial mathematics. A discussion of the solutions was introduced for the linear Black-Scholes model with the European options (Call and Put) analytically and numerically ,with and without transformation to heat equation .Comparisons of the presented approximation solutions of these models with the exact solution were achieved.

KEYWORDS: Linear Black-Scholes Equation, American and European Options, Genetic Algorithm

1. INTRODUCTION

Many mathematical models have been proposed to describe the evolution of the value of financial derivatives. Financial models were generally formulated in terms of stochastic differential equations, [1]. Linear Black-Scholes equation the model was developed by Fischer Black and Myron Scholes [2], as an important equation in the financial mathematics.

$$0 = V_t + \frac{1}{2}\sigma^2 V_{SS} + rSV_S - rV \tag{1}$$

Where $S := S(t) > 0$ and $t \epsilon (0, T)$, provides both the price for a European option and a hedging portfolio that replicates the option assuming that, [4].

- The price of the asset price or underlying derivative $S(t)$ follows a Geometric Brownian motion $W(t)$, meaning that S satisfies the following stochastic differential equation (SDE).

  $dS(t) = \mu S(t)dt + \sigma S(t)dW(t).$

- The trend or drift μ (measures the average rate of growth of the asset price), the volatility σ (measures the standard deviation of the returns) and the riskless interest rate r are constant for $0 \le t \le T$ and no dividends are paid in that time period.

Equation (1) can be transformed into the heat equation to find the price of the option analytically, [1]. In this research, the application of the accelerated Genetic Algorithm method was taken into account to solve the linear Black–Scholes model for European options. Studying (1) for an American options would be redundant, since the value of an American options equals the value of a European options if no dividends are paid and the volatility is constant, [6], [7], [8], [9], [10].

2. THE GENETIC ALGORITHMS

The principles of genetic algorithm are discussed in previous paper [15]. Where The components of the genetic algorithm, [11, 12] are:

- The initial population.
- The fitness function.
- The genetic operators.

2.1. Evolutionary Algorithm Used for Coding Production

To produce fitness function we used Backus-Naur grammar form (BNF), [13, 14, 15].

Where the grammar into as the following:

- Reading an element from the chromosome (with value V).
- Selecting the rule according to the scheme:

Rule = V mod NR (1)

Where NR is the number of rules for the specific non-terminal symbol And the grammar shown in Table 1 below. If the chromosome *g = [7 9 4 14 28 10 12 2 17 15 6 11 10 24 11]*. Table 2 show how the function is produced by the grammar. Thus the grammatical evaluation can found in, [16, 17, 18].

Table 1: The Grammar of the Proposed Method

S::=<expr>	
<expr> ::= <expr> <op> <expr>	(0)
| (<expr>)	(1)
| <func> (<expr>)	(2)
|<digit>	(3)
|x	(4)
|y	(5)
|z	(6)
<op> ::= +	(0)
| -	(1)
| *	(2)
| /	(3)
<func> ::= sin	(0)
|cos	(1)
|exp	(2)
|log	(3)
<digit> ::= 0	(0)
| 1	(1)
| 2	(2)
| 3	(3)
| 4	(4)
| 5	(5)
| 6	(6)
| 7	(7)
| 8	(8)
| 9	(9)

Table 2: Illustrate Example of Program Construction

String	Chromosome	Operation
<expr>	*7 2 9 4 12 2 20 12 31 4*	*7mod7=0*
<expr><op><expr>	*2 9 4 12 2 20 12 31 4*	*2mod 7=2*
Fun(<expr>)< op><expr>	*9 4 12 2 20 12 31 4*	*9mod4=1*
cos(<expr>)< op><expr>	*4 12 2 20 12 31 4*	*4mod7=4*
cos(x)<op><expr>	*12 2 20 12 31 4*	*12mod4=0*
cos(x) + <expr>	*2 20 12 31 4*	*2mod7=2*
cos(x) + Fun(<expr>)	*20 12 31 4*	*20mod4=0*
cos(x) + sin(<expr>)	*12 31 4*	*12mod7=5*
cos(x) + sin(y)		

2.2. Technique of the Proposed Method

The proposed method has the following phases:

- Initialization.
- Fitness evaluation.
- Genetic operations.
- Termination control.

2.2.1. Initialization

The value of mutation rate and selection rate are stated, [13, 15]. The initialization of every chromosome is performed by randomly selecting an integer for every element of the corresponding vector.

2.2.2. Fitness Evaluation

Expressing the Partial differential equation in the following form:

$$f\left(x, y, \frac{\partial u}{\partial x}(x, y), \frac{\partial u}{\partial y}(x, y), \frac{\partial^2 u}{\partial x^2}(x, y), \frac{\partial^2 u}{\partial y^2}(x, y)\right) = 0\ , x \in [x_0, x_1]\ , y \in [y_0, y_1]$$

The associated boundary conditions are expressed as:

$$u(x_0\ , y\ ) = f_0(y)\ , u(x_1\ , y\ ) = f_1(y)\ , u(x\ \ , y_0) = g_0(y)\ , u(x\ \ , y_1) = g_1(y)$$

The steps for the fitness evaluation of the population are the following:

- Choose N^2 equidistant points in the box $[x_0, x_1] \times [y_0, y_1]$, N_x equidistant points on the boundary at $x = x_0$ and at $x = x_1$, N_y equidistant points on the boundary at $y = y_0$ and at $y = y_1$
- For every chromosome i:
  - o Construct the corresponding model $M_i(x ,y)$, expressed in the grammar described earlier.
  - o Calculate the quantity

$$E(M_i) = \sum_{j=0}^{N^2} (f(x_j, y_j, \frac{\partial}{\partial x} M_i(x_j, y_j), \frac{\partial}{\partial y} M_i(x_j, y_j), \frac{\partial^2}{\partial x^2} M_i(x_j, y_j), \frac{\partial^2}{\partial y^2} M_i(x_j, y_j))^2$$

  - o Calculate an associated penalty $P_i\ (M_i)$. The penalty function P depends on the boundary conditions and it has the form:

$$P_1(M_i) = \sum_{j=1}^{N_x}(M_i(x_0, y_j) - f_0(y_j))^2$$

$$P_2(M_i) = \sum_{j=1}^{N_x}(M_i(x_1, y_j) - f_1(y_j))^2$$

$$P_3(M_i) = \sum_{j=1}^{N_y}(M_i(x_j, y_0) - g_0(x_j))^2$$

$$P_4(M_i) = \sum_{j=1}^{N_y}(M_i(x_j, y_1) - g_1(x_j))^2$$

- Calculate the fitness value of the chromosome as:

$$v_i = E(M_i) + P_1(M_i) + P_2(M_i) + P_3(M_i) + P_4(M_i)$$

2.2.3. Genetic Operators

The genetic operators that are applied to the genetic population are the initialization, the crossover and the mutation. A random integer of each chromosome was selected to be in the range [0.255]. The parents are selected via tournament selection, i.e:

- First, create a group of $K >= 2$ randomly selected individuals from the current population.
- The individual with the best fitness in the group is selected, the others are discarded.

The final genetic operator used is the mutation, where for every element in a chromosome a random number in the range [0, 1] is chosen, [15].

2.2.4. Termination Control

Creating new generation required for application genetic operators to the population in order to find the best chromosome having better fitness or whenever the maximum number of generations was obtained.

3. TECHNICAL OF THE ACCELERATED METHOD

To make the method is faster to arrived the exact solution of the partial differential equations by the following:

- Insert the boundary conditions of the problem as a part of chromosomes in the our population of the problem, the algorithm gives the exact solution or approximate solution in a few generations.
- Insert a part of exact solution (or particular solution) as a part of a chromosome in the population, find the algorithm that gives an exact solution in a few generations.
- Insert the vector of exact solution (if exist) as a chromosome in the our population of the problem, the algorithm gives the exact solution in the first generation.

4. EXACT SOLUTION OF THE LINEAR BLACK – SCHOLES EQUATION

To solved equation (1) in closed form [19, 20]:

Let $x = \ln\left(\frac{S}{K}\right)$, $t = T - \frac{\tau}{\sigma^2/_2}$, $and\ v(x,\tau) = \frac{1}{K}V(S,t)$.

Now

$\frac{\partial V}{\partial t} = \frac{\partial V}{\partial \tau}\frac{\partial \tau}{\partial t} + \frac{\partial V}{\partial S}\frac{\partial S}{\partial t} = -K\frac{\sigma^2}{2}\frac{\partial v}{\partial \tau}$,

$\frac{\partial V}{\partial S} = K\frac{\partial v}{\partial S} = \frac{K}{S}\frac{\partial v}{\partial x}$

And $\frac{\partial^2 V}{\partial S^2} = K\frac{\partial^2 v}{\partial S^2} = \frac{K}{S^2}[\frac{\partial^2 v}{\partial x^2} - \frac{\partial v}{\partial x}]$

Substituting these derivatives in (1) one gets

$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + \left(\frac{2r}{\sigma^2} - 1\right)\frac{\partial v}{\partial x} - \frac{2r}{\sigma^2}v$

Letting $\frac{2r}{\sigma^2} = \theta$ then the above equation can be written as :

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (\theta - 1)\frac{\partial v}{\partial x} - \theta v \qquad (2)$$

Now letting $\lambda = \frac{1}{2}(\theta - 1)\,, v = \frac{1}{2}(\theta + 1) = \lambda + 1$

To get $v^2 = \lambda^2 + \theta$

And then

$v(x,\tau) = e^{-\lambda x - v^2\tau}u(x,\tau)$,

Therefore;

$\frac{\partial v}{\partial \tau} = e^{-\lambda x - v^2\tau}(-v^2)u(x,\tau) + \frac{\partial u}{\partial \tau}e^{-\lambda x - v^2\tau}u(x,\tau) = e^{-\lambda x - v^2\tau}u(x,\tau)[(-v^2)u + \frac{\partial u}{\partial \tau}]$

$\frac{\partial v}{\partial x} == e^{-\lambda x - v^2\tau}u(x,\tau)[-\lambda u + \frac{\partial u}{\partial x}]$

$\frac{\partial^2 v}{\partial x^2} = e^{-\lambda x - v^2\tau}[\lambda^2 u - 2\lambda\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}]$

Inserting these into equation (2) and dividing by $e^{-\lambda x - v^2\tau}$, obtaining:

$$u_\tau = u_{xx} \qquad (3)$$

The initial and boundary conditions for the European Call and Put options are respectively

$u(x,0) = (e^{(\lambda+1)x} - e^{\lambda x})^+ \; as \; x\epsilon R,$

$u(x,\tau) = 0 \; as \; x \to -\infty$

$u(x,\tau) = (e^{(\lambda+1)x + v^2\tau} - e^{\lambda x + \lambda^2\tau}) \quad as \; x \to \infty$

And

$u(x,0) = (e^{\lambda x} - e^{(\lambda+1)x})^+ \; as \; x\epsilon R,$

$u(x,\tau) = e^{\lambda^2 x + \lambda\tau} \; as \; x \to -\infty$

$u(x,\tau) = 0 \; as \; x \to \infty$

Thus the Black-Scholes equation reduces to the heat equation. Now considering the problem (3) with initial condition:

For Call Option,

$$u(x,0) = g(x) = \max\{(e^{(\lambda+1)x} - e^{\lambda x}), 0\},$$

And for Put Option

$$u(x,0) = g(x) = \max\{(e^{\lambda x} - e^{(\lambda+1)x}), 0\},$$

Applying the Fourier transformation with respect to *x* in equation (3) where it was solved in [22] yields;

Call Option

$$V(S,t) = Kv(x,\tau) = Ke^{-\lambda x - v^2\tau}u(x,\tau)$$

Therefore

$$V_c(S,t) = S\emptyset(d1) - Ke^{-r(T-t)}\emptyset(d2)$$

Where

$$d1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right).(T-t)}{\sigma\sqrt{T-t}}$$

And

$$d2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right).(T-t)}{\sigma\sqrt{T-t}} = d1 - \sigma\sqrt{T-t}$$

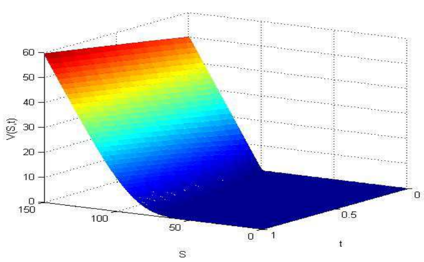
This gives the values for the European Call option where *Φ(d)* the standard normal distribution of the function is *d*. *V(S,t)* can be seen in Figure 1. The corresponding parameters are *K = 100, σ =0.2, r =0 .1, T = 1 year, dt =0 .001.*

Put Option

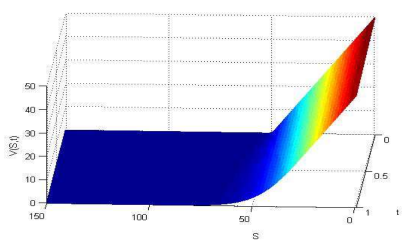
A similar calculation for the European Put option and the pay-off function for the Put option in absence of dividend is

$$V_P(S,t) = Ke^{-r(T-t)}\emptyset(-d2) - S\emptyset(-d1)$$

Figure 1 is related to the corresponding parameters *K = 100, σ =0.2 , r =0 .1, T = 1 year , dt =0 .001.*



(a)



(b)

Figure 1: Exact solutions of Eq. (1), (a) Call Option, (b) Put Option

5. APPLICATIONS OF ACCELERATED GENETIC ALGORITHM

To find numerical solutions of Linear Black-Scholes model and the transformed model we applied the accelerated genetic algorithm method. The crossover rate was set to 75% (that is, replication rate equal to 25%) and mutation rate was set to 1%. Each experiment to find the numerical solution was performed 20 times. The population size was set to 100 and the length of each chromosome to 50. The size of the population is a critical parameter. The generations number set to 100 iterations for each experiment. The function (*randi)* in Matlab R2010a used to generate the initial population.

5.1. Numerical Solution by Accelerated Genetic Algorithm of Transformed Equation

In this section an attempt was made to solve the model numerically. The accelerated genetic algorithm scheme was recalled to solve the transform problem; the heat equation. Then backward substitution reduced the solution of equation (1).

Call Option

Considering the model

$$u_\tau = u_{xx}\ , x \in R\ , 0 \leq \tau < \hat{T}$$

With the initial and boundary conditions for call option as

$$u(x,0) = (e^{(\lambda+1)x} - e^{\lambda x})^+ \ as\ x\epsilon R,$$

$$u(x,\tau) = 0\ as\ x \to -\infty$$

$$u(x,\tau) = (e^{(\lambda+1)x+v^2\tau} - e^{\lambda x+\lambda^2\tau}) \quad as\ x \to \infty$$

The approximate solutions was obtained as:

$u(x,\tau) = e^{-\tau} sinx$ at generation 20

And

$Gp4(x,\tau) = (x + xe^{-\tau})/2$ at generation 4 , as a trail solutions.

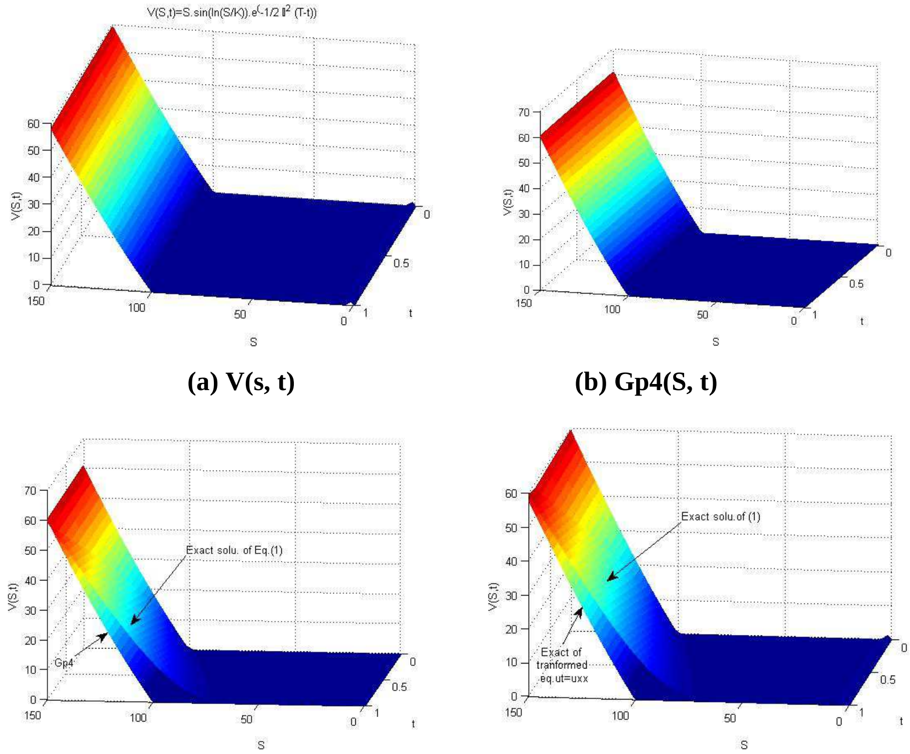
Then, the approximation solutions of (1) are:

$$V(S,t) = S.\sin\,(\ln\left(\frac{S}{K}\right).e^{-\frac{1}{2}\sigma^2(T-t)}$$

And

$$Gp4(S,t) = 0.5S(\ln\left(\frac{S}{K}\right) + \ln\left(\frac{S}{K}\right)e^{-\frac{1}{2}\sigma^2(T-t)})$$

These solutions and comparisons of them with the exact solution of eq. (1) shown in Figure 2:



(a) V(s, t) **(b) Gp4(S, t)**

(c) Exact Solu. of (1) and V(S, t) **(d) Exact Solu. of (1) and Gp4**

Figure 2: Trail Solutions and There Comparisons of them with the Exact Solution of Eq. (1) (call Option)

Put Option

The related model is;

$u_\tau = u_{xx}\ , x \in R\ , 0 \le \tau < \hat{T}$

With the initial and boundary conditions for Put option as

$u(x,0) = (e^{\lambda x} - e^{(\lambda+1)x})^+\ as\ x\epsilon R,$

$u(x,\tau) = e^{\lambda^2 x + \lambda\tau}\ as\ x \to -\infty$

$u(x,\tau) = 0\ as\ x \to \infty$

And the approximate solutions are;

$u(x,\tau) = -1 + \frac{1}{4}e^{-x}(e^{\tau/8} + 3e^{\tau/4})$; at generation 20 , and

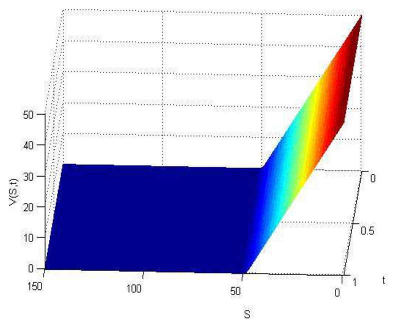
$Gp4(x,\tau) = -1 + e^{-x-\tau}$; at generation 4

The back substitution of the variables transformation can be used to get the solution of a problem related to the partial differential equation (1). The numerical solutions are:

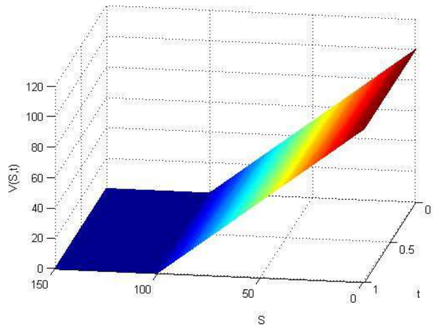
$V(S,t) = -S + \frac{K}{4}(e^{(\frac{1}{2}\sigma^2(T-t))/8} + 3e^{(\frac{1}{2}\sigma^2(T-t))/4}$; and

$Gp4(S,t) = -S + Ke^{-\frac{1}{2}\sigma^2(T-t)}$

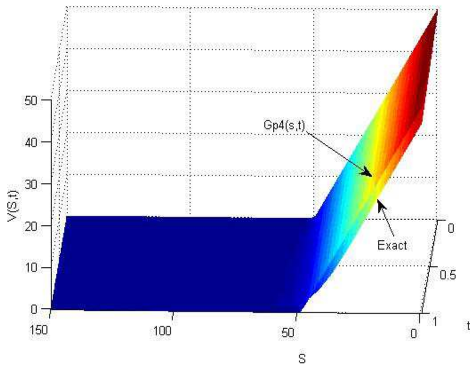
These solutions and comparisons of them with the exact solution of eq. (1) shown in Figure 3:



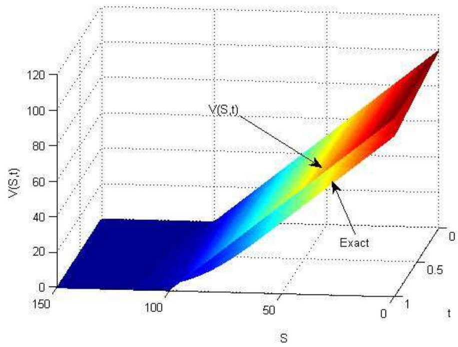
(a) Gp4(S, t)



(b) V(S, t)



(c) Exact Solu. of (1) and Gp4



(d) Exact Solu. of (1) and V(S, t)

Figure 3: Trail Solutions and Comparisons with the Exact Solution of eq. (1); Put Option

5.2. Numerical Solution of Original Equation by Accelerated Genetic Algorithm

In this section a trial was achieved to solve the original model numerically. The use of accelerated genetic algorithm scheme was recalled to solve the original equation (1).

$$0 = V_t + \frac{1}{2}\sigma^2 V_{SS} + rSV_S - rV$$

For call and put options with boundary and initial conditions

$$V(0,t) = 0 \; for \; 0 \leq t \leq T$$

$$V(S,t) \sim S - Ke^{-r(T-t)} \; as \; S \to \infty$$

$$V(S,T) = (S-K)^+ \; for \; 0 \leq S < \infty$$

And

$$V(0,t) = Ke^{r(T-t)} \; for \; 0 \leq t \leq T$$

$$V(S,t) \sim 0 \; as \; S \to \infty$$

$$V(S,T) = (K-S)^+ \; for \; 0 \leq S < \infty$$

For Call Option

$$Gp5(S,t) = V(S,t) = S - 100e^{-\frac{1}{9}(T-t)}$$

as a trail solution at generation 5 , as well as

$$Gp7(S,t) = V(S,t) = S - 81e^{-0.02(T-t)}$$

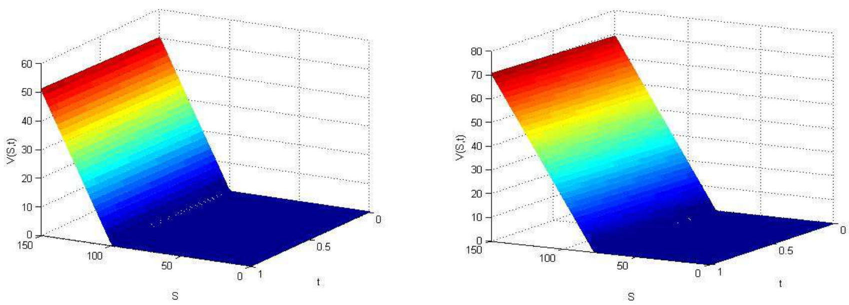
as the trail solution at generation 7.

For ***put option*** the above solution satisfied such that:

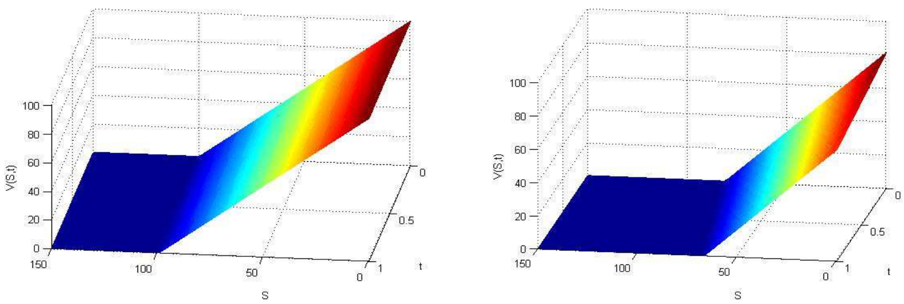
$$Gp5'(S,t) = V(S,t) = 100e^{-\frac{1}{9}(T-t)} - S$$

And $Gp7'(S,t) = V(S,t) = 81e^{-0.02(T-t)} - S$

These solutions are represented by in Figure 4.



(b) Gp5(S, t) (Call Option) **(b) Gp7(S, t) (Call Option)**



(c) Gp5'(S, t) Put Option **(d) Gp7'(S, t) Put Option**

Figure 4: Numerical Solutions of Original Equation without Transformation

6. COMPARISON THE STUDY

Comparison all solutions for European Options (call and put) corresponding parameters *K = 100, σ =0. 2 , r =0 .1, T = 1 year , S=0-150* for both call and put options:

6.1. Call Option

In Table 3 and Table 4 below we show that the errors between these solutions, and the difference between these errors are shown Figure 5-a.

Table 3: Comparison of Solutions for European Call Option (Exact Solution with Numerical Solutions of Transformed Eq)

S	t	Exact Solu. of Eq.(1)	V(S, t) of Transformed Eq.(3)	Gp4(S, t) for Transformed Eq.(3)	Error = Abs(Exact-V(S, t))	Error = Abs(Exact-Gp4)
0	1	0	0	0	0	0
7.8947	0.9473	0	0	0	0	0
15.7894	0.8947	0	0	0	0	0
23.6842	0.8421	8.0178e-14	0	0	8.02E-14	8.02E-14
31.5789	0.7894	1.1445e-09	0	0	1.14E-09	1.14E-09
39.4736	0.7368	5.9960e-07	0	0	6E-07	6E-07
47.3684	0.6842	4.9382e-05	0	0	4.94E-05	4.94E-05
55.2631	0.6315	0.0012	0	0	0.0012	0.0012
63.1578	0.57894	0.0151	0	0	0.0151	0.0151
71.0526	0.5263	0.1029	0	0	0.1029	0.1029
78.9473	0.4736	0.4636	0	0	0.4636	0.4636
86.8421	0.4210	1.5177	0	0	1.5177	1.5177
94.7368	0.3684	3.8485	0	0	3.8485	3.8485
102.6315	0.3157	7.9136	2.6488	2.6575	5.2648	5.2561
110.5263	0.2631	13.6821	10.9854	11.0328	2.6967	2.6493
118.4210	0.2105	20.5733	19.8432	19.9801	0.7301	0.5932
126.3157	0.1578	27.8843	29.1493	29.4627	1.265	1.5784
134.2105	0.1053	35.2576	38.8408	39.4485	3.5832	4.1909
142.1052	0.0526	42.6302	48.8626	49.9092	6.2324	7.279
150	0	50	59.1669	60.8197	9.1669	10.8197

Table 4: Comparison of Solutions for European Call Option (Exact Solution with Numerical Solutions of Eq. 1)

S	t	Exact Solu. of Eq.(1)	Gp5(S,t) of Eq.(1) by Our Method	Gp7(S,t) of Eq.(1) by Our Method	Error = Abs(Exact-Gp5)	Error = Abs(Exact-Gp7)
0	1	0	0	0	0	0
7.8947	0.9473	0	0	0	0	0
15.7894	0.8947	0	0	0	0	0
23.6842	0.8421	8.0178e-14	0	0	8.02E-14	8.02E-14
31.5789	0.7894	1.1445e-09	0	0	1.14E-09	1.14E-09
39.4736	0.7368	5.9960e-07	0	0	6E-07	6E-07
47.3684	0.6842	4.9382e-05	0	0	4.94E-05	4.94E-05
55.2631	0.6315	0.0012	0	0	0.0012	0.0012
63.1578	0.57894	0.0151	0	0	0.0151	0.0151
71.0526	0.5263	0.1029	0	0	0.1029	0.1029
78.9473	0.4736	0.4636	0	5.2687	0.4636	4.8051
86.8421	0.4210	1.5177	0	12.3838	1.5177	10.8661
94.7368	0.3684	3.8485	0	19.4906	3.8485	15.6421
102.6315	0.3157	7.9136	2.9468	26.5891	4.9668	18.6755
110.5263	0.2631	13.6821	10.7891	33.6792	2.893	19.9971
118.4210	0.2105	20.5733	18.6313	40.7607	1.942	20.1874
126.3157	0.1578	27.8843	26.4735	47.8337	1.4108	19.9494
134.2105	0.1053	35.2576	34.3157	54.8979	0.9419	19.6403
142.1052	0.0526	42.6302	42.1578	61.9534	0.4724	19.3232
150	0	50	50	69	0	19

6.2. Put Option

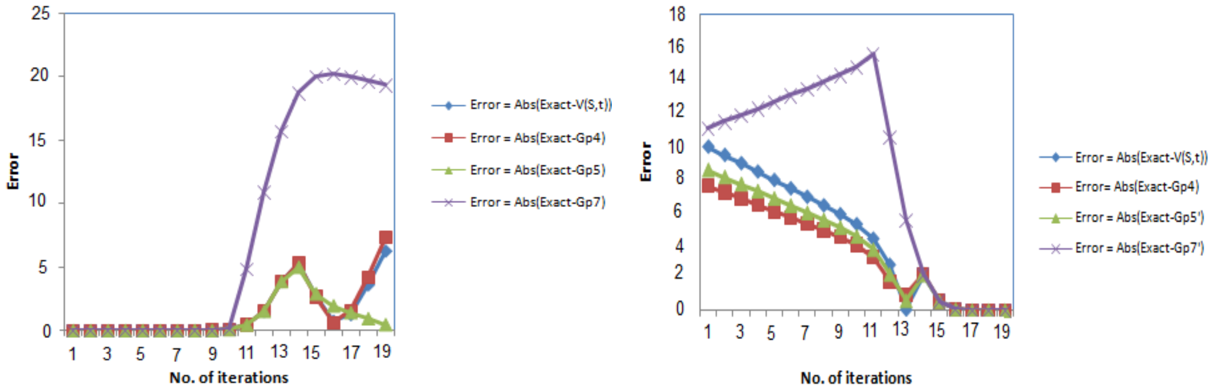
In Table 5 and Table 6 below we show that the errors between these solutions, and the difference between these errors are shown Figure 5-b.

Table 5: Comparison of Solutions for European Put Option (Exact Solution with Numerical Solutions of Transformed Eq)

S	t	Exact Solu. of Eq.(1)	V(S, t) of Transformed Eq.(3)	Gp4(S ,t) for Transformed Eq.(3)	Error = Abs(Exact-V(S, t))	Error= Abs(Exact-Gp4)
0	1	90.4837	100.4385	98.0198	9.9548	7.5361
7.8947	0.9473	83.0664	92.5206	90.2283	9.4542	7.1619
15.7894	0.8947	75.6517	84.6027	82.4369	8.951	6.7852
23.6842	0.8421	68.2395	76.6849	74.6456	8.4454	6.4061
31.5789	0.7894	60.8299	68.7670	66.8545	7.9371	6.0246
39.4736	0.7368	53.4228	60.8492	59.0634	7.4264	5.6406
47.3684	0.6842	46.0183	52.9313	51.2724	6.913	5.2541
55.2631	0.6315	38.6176	45.0135	43.4816	6.3959	4.864
63.1578	0.5789	31.2321	37.0957	35.6908	5.8636	4.4587
71.0526	0.5263	23.9232	29.1779	27.9002	5.2547	3.977
78.9473	0.4736	16.8899	21.2600	20.1097	4.3701	3.2198
86.8421	0.4210	10.5525	13.3422	12.3193	2.7897	1.7668
94.7368	0.3684	5.4945	5.4244	4.5290	0.0701	0.9655
102.6315	0.3157	2.1735	0	0	2.1735	2.1735
110.5263	0.2631	0.5585	0	0	0.5585	0.5585
118.4210	0.2105	0.0689	0	0	0.0689	0.0689
126.3157	0.15789	0.0020	0	0	0.002	0.002
134.2105	0.1052	1.9454e-06	0	0	1.95E-06	1.95E-06
142.1052	0.0526	0	0	0	0	0
150	0	0	0	0	0	0

Table 6: Comparison of Solutions for European Put Option (Exact Solution with Numerical Solutions of Eq. (1))

S	t	Exact Solu. of Eq.(1)	Gp5'(S, t) of Eq.(1) by Our Method	Gp7'(S, t) of Eq.(1) by Our Method	Error = Abs(Exact-Gp5')	Error = Abs(Exact-Gp7')
0	1	90.4837	99.0049	79.396	8.5212	11.0877
7.8947	0.9473	83.0664	91.1623	71.5849	8.0959	11.4815
15.7894	0.8947	75.6517	83.3197	63.7739	7.668	11.8778
23.6842	0.8421	68.2395	75.4772	55.963	7.2377	12.2765
31.5789	0.7894	60.8299	67.6346	48.1521	6.8047	12.6778
39.4736	0.7368	53.4228	59.7921	40.3413	6.3693	13.0815
47.3684	0.6842	46.0183	51.9497	32.5307	5.9314	13.4876
55.2631	0.6315	38.6176	44.1072	24.7201	5.4896	13.8975
63.1578	0.5789	31.2321	36.2648	16.9096	5.0327	14.3225
71.0526	0.5263	23.9232	28.4224	9.0992	4.4992	14.824
78.9473	0.4736	16.8899	20.58	1.2888	3.6901	15.6011
86.8421	0.4210	10.5525	12.737	0	2.1845	10.5525
94.7368	0.3684	5.4945	4.8954	0	0.5991	5.4945
102.6315	0.3157	2.1735	0	0	2.1735	2.1735
110.5263	0.2631	0.5585	0	0	0.5585	0.5585
118.4210	0.2105	0.0689	0	0	0.0689	0.0689
126.3157	0.15789	0.0020	0	0	0.002	0.002
134.2105	0.1052	1.9454e-06	0	0	1.95E-06	1.95E-06
142.1052	0.0526	0	0	0	0	0
150	0	0	0	0	0	0



(a) Call Option **(b) Put Option**

Figure 5: Difference Errors (a) Call Option, (b) Put Option

7. CONCLUSIONS

In this study the accelerated genetic algorithm method was used to obtaining a wide class of approximations solutions for the linear Black-Scholes model with transformed to heat equation and without transformed, and then choosing the best numerical solution among these solutins. One can noted that this method has general utility for applications. Finally, the accelerated Genetic Algorithm has convenient used for approximating the analytical solutions for this model.

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